Linear Algebra I

19/12/2013, Thursday, 14:00-16:00

You are **NOT** allowed to use any type of calculators.

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1 (6+6+6+6+6=30 pts) **Linear equations**

Let

$$A = \begin{pmatrix} 1 & 2 & -1 & 0 & 5 \\ 2 & 2 & 0 & -2 & 10 \\ -1 & 0 & -1 & 2 & -5 \\ 1 & 8 & -7 & 6 & -1 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 0 \\ 0 \\ \alpha \\ -6 \end{pmatrix}$$

where α is a real number. Consider the linear equation $Ax = b$.

(a) Determine the *lead* and *free* variables.

(b) Determine all values of α for which the equation has *infinitely many solutions.*

(c) Determine all values of α for which the equation is *inconsistent.*

(d) Determine all values of α for which the equation has *exactly one solution.*

(e) Find the solution set of the equation for $\alpha = 0$.

2 (15 pts) **Inverse matrix**

Is the matrix

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 4 & 9 & 16 \end{pmatrix}$$

nonsingular? If so, find its inverse.

3 (20 pts) **Determinants**

Let a, b, and c be real numbers. Show that

$$\det \begin{pmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{pmatrix} = (a-b)(b-c)(c-a).$$

4 (5+20=25 pts) **Vector spaces**

Consider the vector space P_4.

(a) Show that $\{\, p \in P_4 \mid p(0) = 1 \,\}$ is *not* a subspace of P_4.

(b) Show that $\{\, p \in P_4 \mid p(1) = 0 \,\}$ is a subspace of P_4. Find a basis for this subspace. What is the dimension of this subspace?

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10 pts free